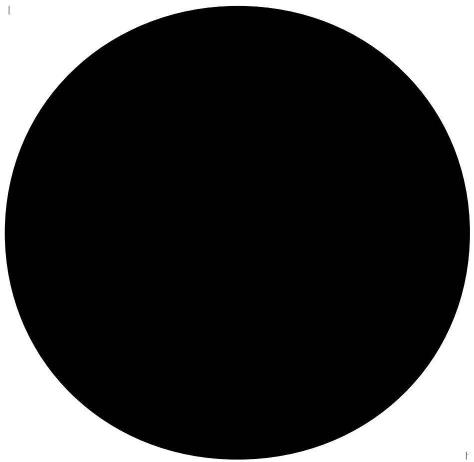
Irrelevant deformations and holography

Monica Guica

IphT, CEA Saclay

Motivation

- black hole entropy



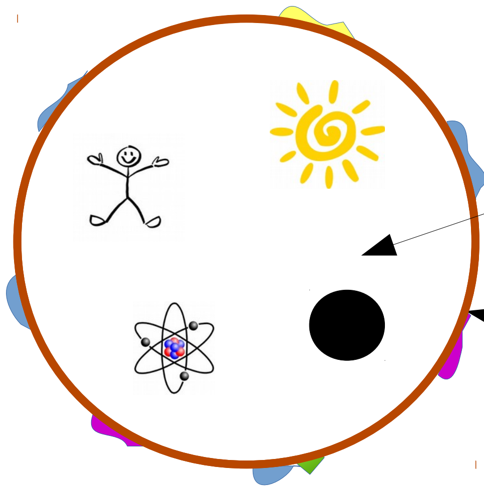
$$S_{BH} = \frac{\mathcal{A}_H}{4G}$$

universal

Introduction

- black hole entropy

Holography



Gravitational theory

d+1 dimensions

Field theory (no gravity)

d dimensions

✔ usual Quantum Mechanics

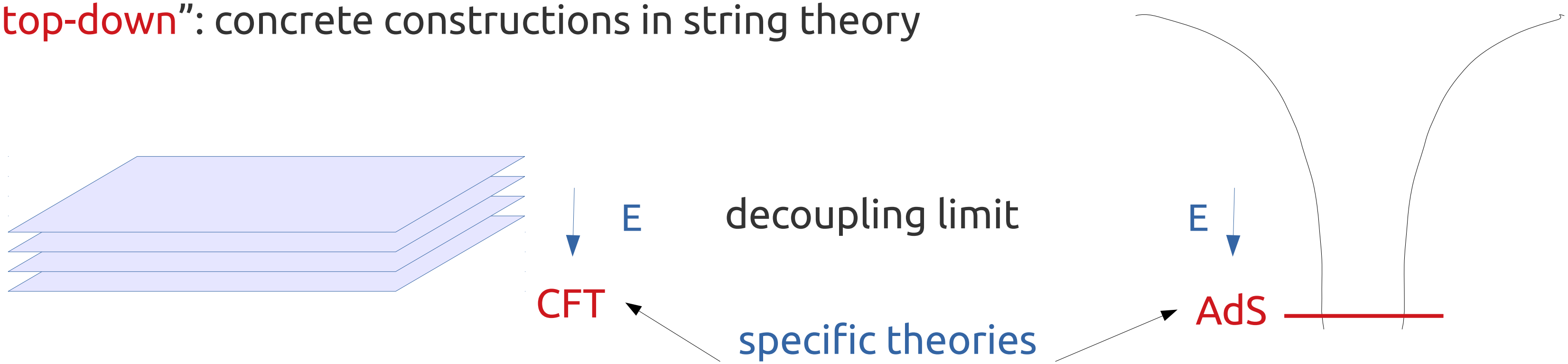
$$S_{BH} = \frac{\mathcal{A}_H}{4G}$$

universal

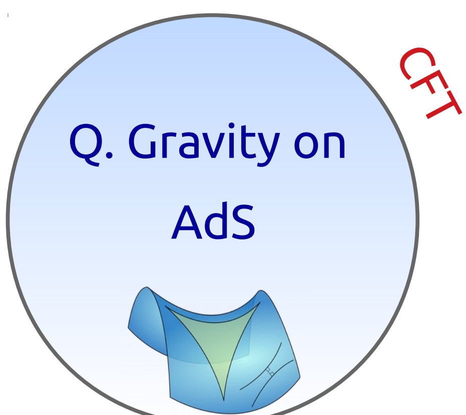
The AdS/CFT correspondence

- AdS/CFT correspondence: two approaches

I. "top-down": concrete constructions in string theory



II. "bottom-up": universalist approach



- $\forall$ CFT_d with large N (large gap) → gravity in AdS_{d+1}
- symmetries → asymptotic symmetries (Virasoro 3d)
- correlation functions → scattering
- axiomatic description of CFTs, even at strong coupling

Beyond AdS/CFT

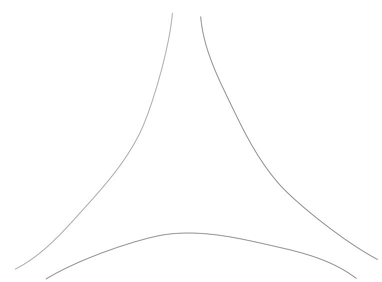
- non-AdS holography : **hard** → no concrete examples in string theory for asympt. flat, de Sitter, etc.

- infer properties of dual QFT from spacetime: symmetries, thermodynamics, correlation functions

e.g. celestial holography programme

symmetries

(asymptotic symmetries)



entropy

(black holes)

correlation functions

(scattering)

Motivation from non-AdS holography

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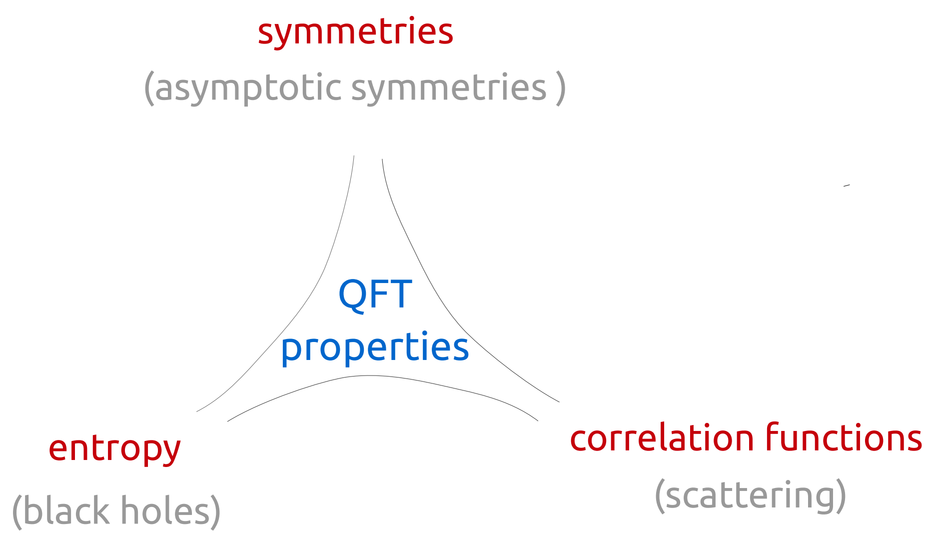

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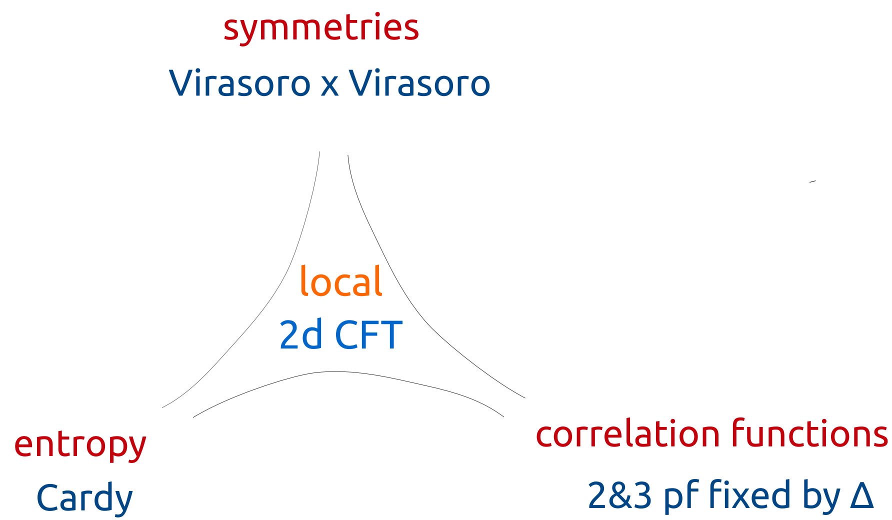

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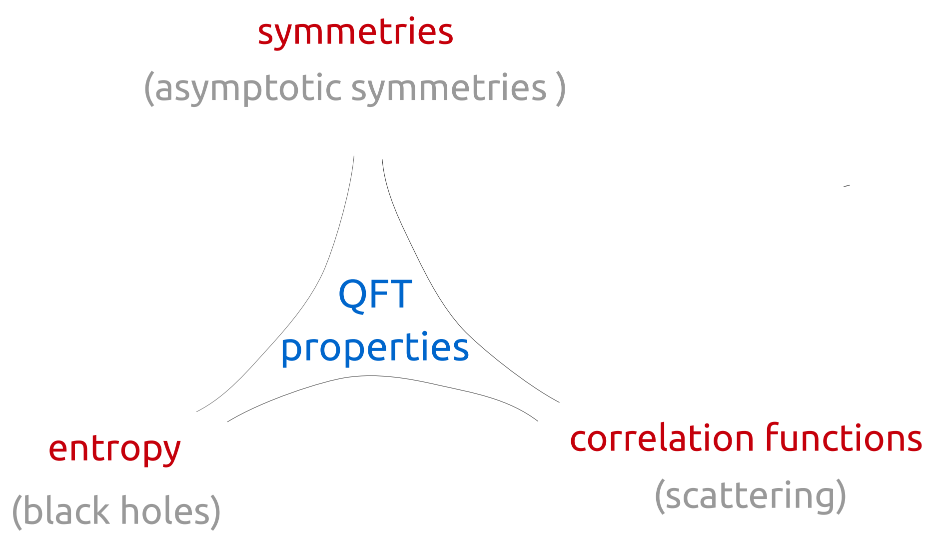
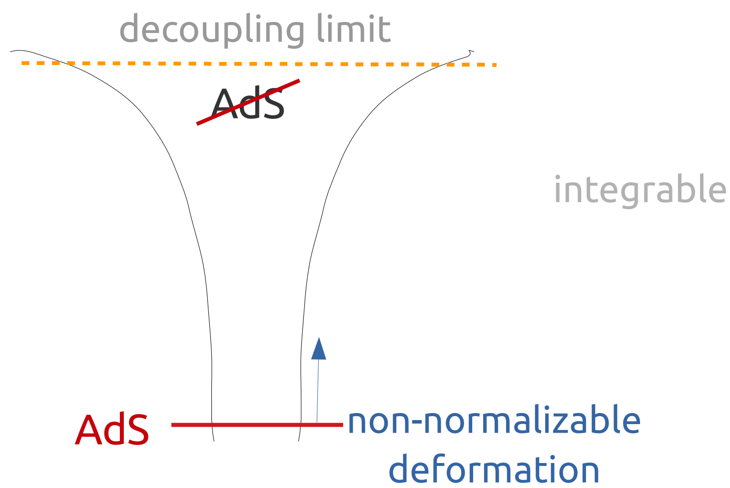


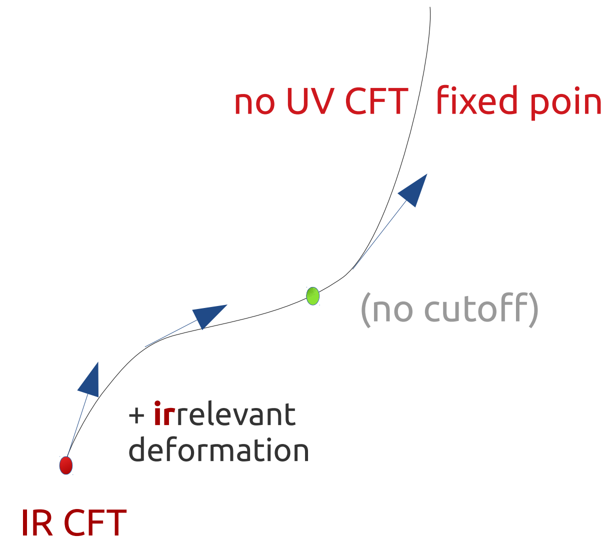
- this data is hard to assemble into a consistent whole w/o knowing the basic QFT structure/properties

- dictionary also not exactly known ; idem for the boundary conditions

- having an in principle independent QFT description is valuable

Irrelevant flows and non-AdS holography





- holographic dual ↔ finely-tuned irrelevant flow → (non-local) QFT

- ∃ concrete examples of finely tuned irrelevant flows dual to non-AdS backgrounds
- even if intractable, can give information about dimension, where it lives etc.

Plan

I : review some old examples of non-AdS holography in string theory → lessons for “universalist” approaches to certain holographic correspondences

II : exactly solvable irrelevant deformations ($T\bar{T}$ and $J\bar{T}$) of two-dimensional CFTs and their applications to (non)-AdS holography

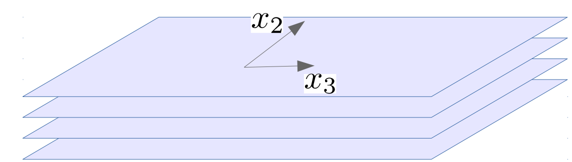
I: String-theoretical examples

Example 1: Non-commutative N=4 SYM

T-duality, shift, T-duality

- D3 branes in a spatial B - field ← induced by **TsT** + decoupling

$$\alpha' \to 0$$
$$B \to \infty$$



- ➡ non-commutative $\mathcal{N}=4$ SYM $\quad \cdot \to \star \quad$ Moyal star product

$$[x^i, x^j] = i\theta^{ij}$$

$$f(x) \star g(x) = e^{\frac{i}{2}\theta^{ij}\frac{\partial}{\partial \xi^i}\frac{\partial}{\partial \zeta^j}} f(x+\xi)g(x+\zeta)|_{\xi=\zeta=0}$$

Seiberg, Witten '99

- planar diagrams: same as in $\mathcal{N}=4$ SYM up to phase factors involving the external momenta

→ free energy (thermodynamics) same as in $\mathcal{N}=4$ SYM

Filk '96

- field redefinition NC $\mathcal{N}=4$ SYM → ordinary $\mathcal{N}=4$ SYM + infinite # of irrelevant operators

with finely - tuned coefficients $\sim \theta^n \mathcal{O}_{4+2n}$

→ UV – completeness, properties of diagrams and thermodynamics mysterious in this picture

Holographic dual

- dual background: obtained via TsT + decoupling

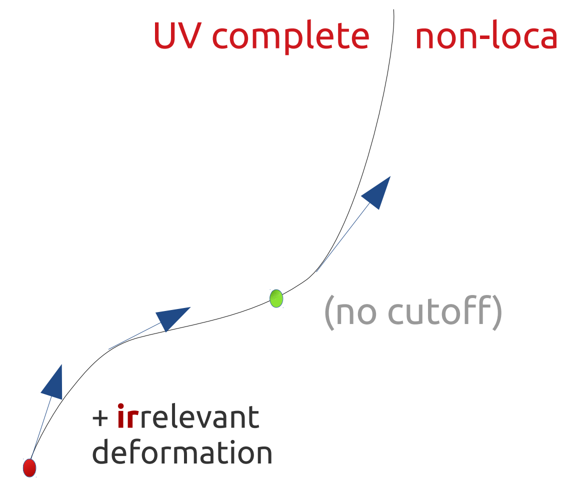
Maldacena, Russo '99

$$ds^2 = gN\alpha' \left[r^2(-dt^2 + dx_1^2) + \frac{r^2}{1+b^2r^4}(dx_2^2 + dx_3^2) + \frac{dr^2}{r^2} + d\Omega_5^2 \right] \qquad e^{2\phi} = \frac{g^2}{1+b^2r^4}$$

- interpolates between $AdS_5 \times S^5$ in the IR → funny asymptotics in the UV (NC SYM)

  → leading irrelevant deformation: dim 6

  → can argue for UV completeness from decoupling limit



- thermodynamics same as $\mathcal{N} = 4$

- correlation functions : non-locality → momentum-space

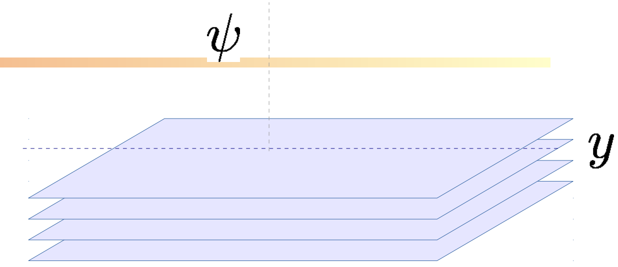
  → gauge-invariant operators : "open Wilson lines" $\Delta x^\mu = \theta^{\mu\nu} k_\nu$

  → match between field theory and gravity

Gross, Hashimoto, Itzhaki '00

Rozali & van Raamsdonk '00

Example 2: Dipole-deformed N=4 SYM



- **TsT** along || and $\perp$ direction to the branes
- star-product deformation of dual field theory : dipole star product

Bergman, Ganor '00

$$\Phi \;\rightarrow\; L^{\mu}_{\Phi} = q_{\Phi}\lambda^{\mu} \qquad\qquad (\Phi_1 \star \Phi_2)(x^{\mu}) = \Phi_1(x^{\mu} - \frac{1}{2}L^{\mu}_2)\,\Phi_2(x^{\mu} + \frac{1}{2}L^{\mu}_1)$$

non-local

dipole length

- planar diagrams unaffected up to a phase → large N free energy unaffected
- Seiberg-Witten map: dipole theory = $\mathcal{N} = 4\;\; SYM$ + infinite number of irrelevant ~~Lorentz~~ operators
- dual gravity background: TsT of original background + decoupling limit

Applications

- holographic modeling of strongly-coupled systems with non-relativistic conformal invariance, a.k.a. AdS/cold atom correspondence

D.T. Son' 08

Adams & Balasubramanian '08

$$ds^2 = -\lambda^2 r^4 (dx^+)^2 + r^2 \left(dx^+ dx^- + \sum_{i=1}^{d-2} dx_i^2 \right) + \frac{dr^2}{r^2}$$

$$x^+ \to c^2 x^+ \,, \quad x^- \to x^-$$
$$x^i \to c\, x^i$$

- Schrödinger$_{d+1}$ backgrounds ⟺ NR CFT$_{d-1}$ codimension 2 holography (symmetries)

- however, certain Schrödinger$_5 \times S^5$ backgrounds ⟺ null dipole–deformed $\mathcal{N}=4$ SYM $\lambda^\mu || \hat{x}^-$

  → $\mathcal{N}=4$ + infinite # of Schrödinger-invariant irrelevant operators
  - non-local along x^-
  - local and Schröd.-invar. along x^+, x^i

- NR CFT : compactify x^- ↔ DLCQ of null dipole theory (additional reduction along non-local direction)

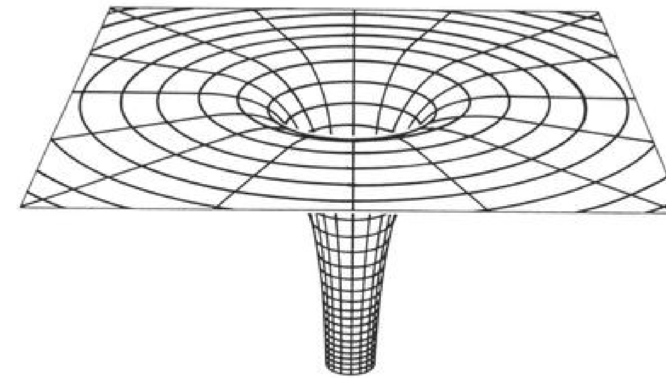
Maldacena, Martelli, Tachikawa '08

- this type of NR CFT has a very special structure

The Kerr / CFT correspondence

- extreme Kerr black hole $GM^2 \simeq J$ e.g. GRS 1915+105

$$ds^2 = 2J\,\Omega^2(\theta)\Big[\underbrace{-r^2dt^2 + \frac{dr^2}{r^2}}_{SL(2,\mathbb{R})_L} + \frac{\sin^2\theta}{\Omega^4(\theta)}(\underbrace{d\phi}_{U(1)_R} + rdt)^2 + d\theta^2\Big]$$



Near Horizon Extreme Kerr

$wAdS_3$

- Kerr/CFT :
  - infinite # of symmetries → Virasoro $c = 12J$
  - Kerr entropy reproduced by Cardy $\approx \frac{1}{2}$ CFT$_2$

- universality (all extremal black holes have Virasoro + entropy match)

- scattering amplitudes look like momentum-space CFT$_2$ correlation functions (on both sides), but

Bredberg, Hartman, Song, Strominger '09

with momentum-dependent conformal dimensions $h_L(\bar{p}), h_R(\bar{p})$

non-local !

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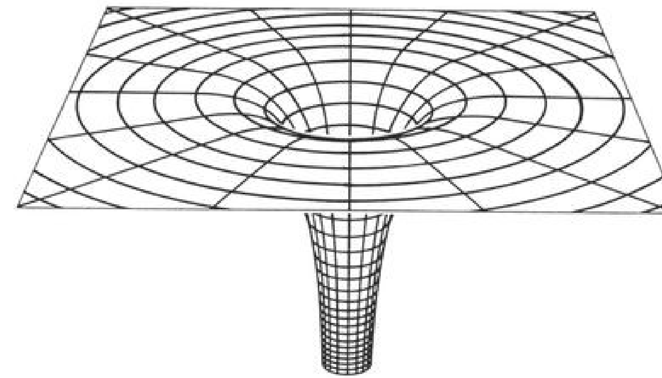

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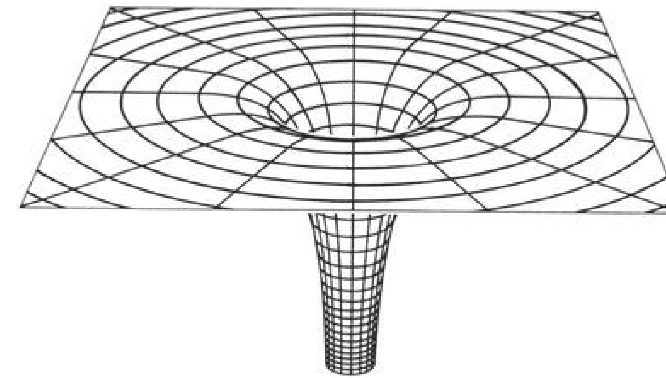

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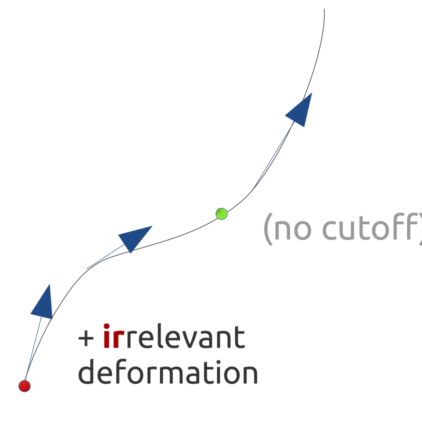
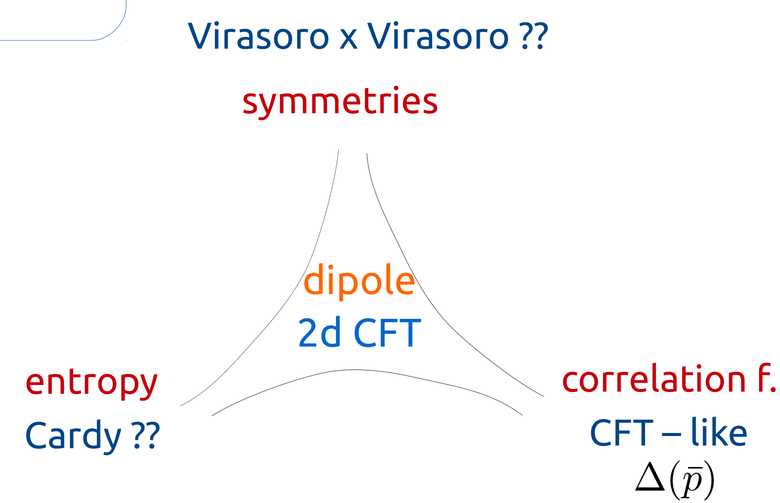
String-inspired interpretation of Kerr/“CFT’’

- warped AdS_3 → finite-temperature versions of $Schr_3$ (NR symmetries: $SL(2,\mathbb{R})_L \times U(1)_R$)
- e.g. TsT of intersecting D1-D5 + flow to IR → not tractable in field theory
- decoupling argument → UV complete theory

$$S_{dipole\,CFT} = S_{CFT} + \lambda \int d^2x\, \mathcal{O}_{(1,2)} + \dots$$

- dipole CFT : = UV-complete 2d QFT : local & conformal on the left
  - non-local on the right
- agrees with non-local features of the correlation functions
- Kerr/“CFT” analysis suggests that, despite non-locality
- hard to check without a concrete example





Assessment of string constructions

- can realise specific examples of non-AdS holography using specific brane constructions

  → very different from standard QFT intuition (non-local, but with very special structure)

- usually tractable when boundary theory is a gauge theory (YM, D3)

- usually not tractable for 3d bulk (intersecting branes)

- very special constructions → not easy to generalize, but still expect similar QFT structure

- simpler, more tractable setups?

Solvable irrelevant deformations of 2d CFTs

Smirnov-Zamolodchikov deformations

- irrelevant deformations of 2d QFTs → bilinears of two (higher spin) conserved currents J^A, J^B
- define

$$\mathcal{O}_{J^A J^B} \ : \quad \lim_{y \to x} \epsilon^{\alpha\beta} J^A_\alpha(x) J^B_\beta(y) = \mathcal{O}_{J^A J^B} + \text{ derivative terms}$$

Zamolodchikov '04

SZ '16

nice factorization properties

- deformation :

$$\frac{\partial S(\mu)}{\partial \mu} = \int d^2x\, \mathcal{O}_{J^A J^B}(\mu)$$

- examples:

$T\bar{T}: \quad J^A_\alpha = T_\alpha{}^A \,, \quad J^B_\beta = T_\beta{}^B \quad (\times \epsilon_{AB})$ (2,2)

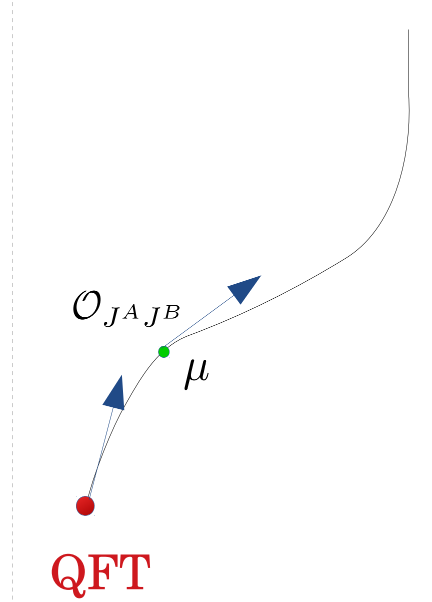
universal

$J\bar{T}: \quad J^A_\alpha = J_\alpha \,, \quad J^B_\beta = T_{\beta\bar{z}}$ ~~Lorentz~~ (1,2)

$SL(2,\mathbb{R})_L \times U(1)_R$

local & conformal — non-local!

- highly tractable : exact finite -size spectrum, S-matrix, preserves integrability
- deformed theory non-local (scale $\mu^{\#}$) but argued UV complete



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same as extremal black holes ←

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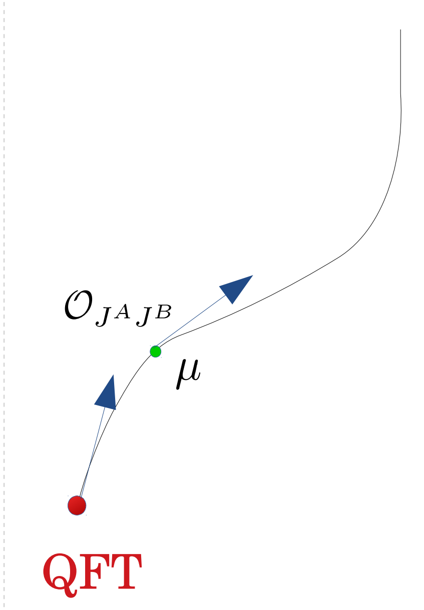

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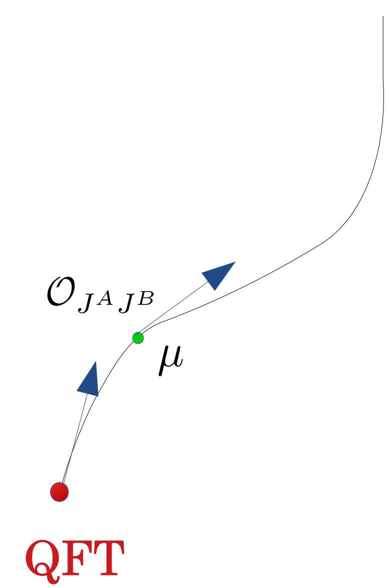

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The “QFT properties’’ of TT – deformed CFTs

$$\partial_\mu |n_\mu\rangle = \mathcal{X}_{T\bar{T}} |n_\mu\rangle \qquad \partial_\mu \tilde{L}^\mu_m = [\mathcal{X}_{T\bar{T}}, \tilde{L}^\mu_m]$$ conserved

MG ‘21

symmetries

Virasoro x Virasoro

$T\bar{T}$ - def CFT

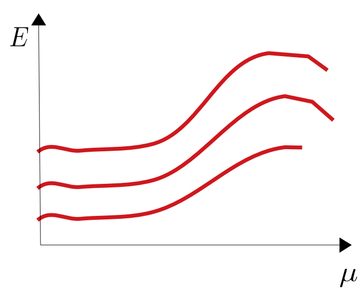
entropy

Hagedorn

correlation functions

$$\langle \mathcal{O}(q)\mathcal{O}(-q)\rangle \sim \int d^2\sigma \, e^{iq\sigma} \frac{\#}{\sigma^{2\Delta+\mu q^2}}$$

Aharony, Barel ‘23



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conserved

MG '21

non-standard

symmetries

Virasoro x Virasoro

despite non-locality

$T\bar{T}$ - def CFT

entropy

Hagedorn

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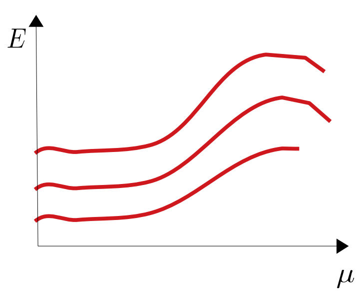

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conserved

field-dependent coordinate

$$\tilde{L}^{cls}_m = (R + 2\mu H_R) \int d\sigma\, f(u) \mathcal{H}_L$$

rescaled

MG, Monten, Tsiares ‘22

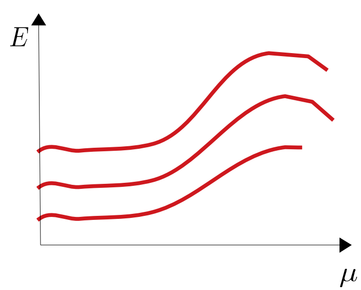
symmetries

Virasoro x Virasoro

T$\bar{\text{T}}$ - def CFT

entropy

Hagedorn



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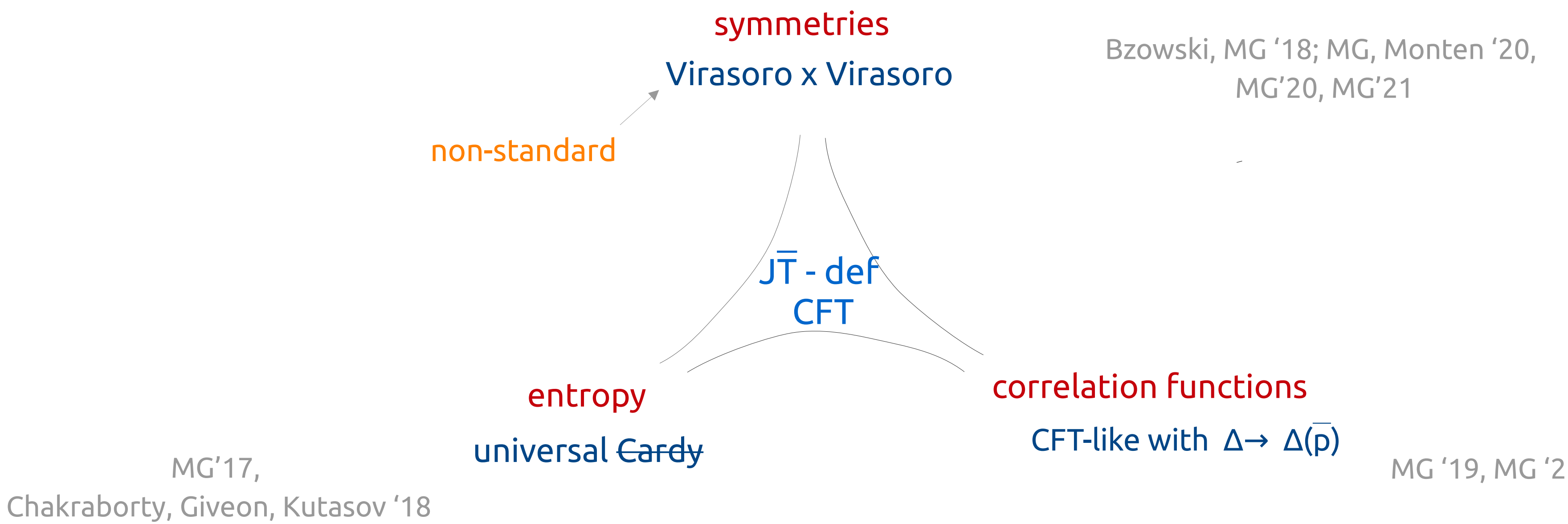
Aharony, Barel ‘23

The “QFT properties” of $J\bar{T}$ – deformed CFTs

- $J\bar{T}$ – deformed CFTs are dipole CFTs $\underbrace{SL(2,\mathbb{R})_L}_{\text{local \& conformal}} \times \underbrace{U(1)_R}_{\text{non-local}}$



- left: flowed Virasoro explicitly different from generator of left conformal transformations

- $\exists$ non-local analogues of primary operators whose correlators are entirely determined by seed CFT

Holographic interpretation

- in AdS/CFT parlance, the Smirnov-Zamolodchikov deformations are double-trace → mixed boundary conditions for the dual fields

  $T\bar{T}$: mixed boundary conditions on the asymptotic metric (FG coefficients)

$$\gamma_{\alpha\beta}(\mu) = g^{(0)}_{\alpha\beta} - \mu g^{(2)}_{\alpha\beta} + \frac{\mu^2}{4} g^{(2)}_{\alpha\gamma} g^{(0)\gamma\delta} g^{(2)}_{\delta\beta} = fixed$$

  pure gravity ~ Dirichlet at $\rho = -\mu$

  $J\bar{T}$: mixed boundary conditions b/w the asymptotic metric and U(1) Chern-Simons gauge field

  ~ Compere-Song-Strominger bnd. cond. in metric sector, but ASG has different interpretation

- 1st instance of mixed bnd. cond. on AdS_3 metric → bulk & boundary have independent definitions

  → precision check of the holographic dictionary

- change bnd. conditions on AdS_3 metric → radical modification of the bnd. theory: local → non-local

- $T\bar{T}$, $J\bar{T}$ $\not\rightarrow$ non-AdS geometry b/c they are double-trace → need single-trace irrelevant deformations

Single-trace $T\bar{T}$ / $J\bar{T}$ – deformed CFTs

- AdS_3/CFT_2 gauge group: S_p (permutations) → consider symmetric product orbifold CFTs $\mathcal{M}^p/S_p$
- standard $T\bar{T}$: double-trace $\sum_i T_i \sum_j \bar{T}_j$
- seed $\mathcal{M}^p/S_p$: single-trace $T\bar{T}$ deformation $\sum_{i=1}^p T_i \bar{T}_i \quad \Rightarrow \quad (T\bar{T}_{def.}\mathcal{M})^p/S_p$
- exact partition function, spectrum, thermodynamics, correlation functions

Apolo, Song ' 23
Chakraborty, Georgescu, MG '23

- can also show Virasoro & fractional Virasoro generators survive, as well as the flowed KdV charges
- the non-linear algebra of the unrescaled symmetry generators is (untwisted sector)

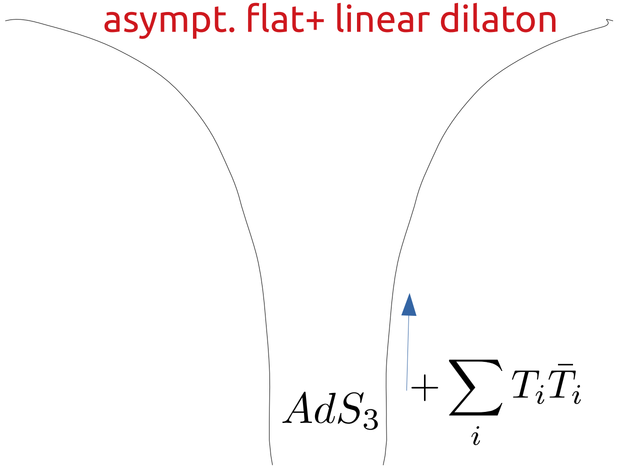
$$[Q_m, Q_n] = (m-n)\sum_i \frac{Q^i_{m+n}}{R+2\mu H^i_R} + (m-n)\sum_i \frac{4\mu^2 H^i_R Q^i_m Q^i_n}{R^i_u R^i_H} + \frac{c}{12} m(m^2-1)\sum_i \frac{1}{(R^i_u)^2}$$

$$R^i_u = R + 2\mu H^i_R$$

$$R^i_H = R + 2\mu H^i$$

- same as double-trace algebra, but with $\mu \to \mu/p$ inside expectation values
- dual to a stringy background

The NS5-F1 system and "single-trace $T\bar{T}$ "



can have black holes

k NS5 and p F1 strings in the NS5 decoupling limit

$g_s \to 0$, α' fixed

p large

UV: Little String Theory

non-gravitational, non-local theory with Hagedorn growth

IR: AdS_3 long strings: descr. by $(\mathcal{M}_{6k})^p/S_p$ SPO

short strings: not ~

- worldsheet σ - model : exactly marginal deformation of the WZW model describing AdS_3 by $J^-\bar{J}^-$

→ dual to CFT source for a $(2,2)$ single-trace operator ~ $\sum_{i=1}^{p} T_i\bar{T}_i$

Giveon, Itzhaki, Kutasov '17

Status of the correspondence

"weak form" : the long string sector of string theory on this background ⟺ single-trace $T\bar{T}$

- spectrum of long string excitations exactly matches single-trace $T\bar{T}$ spectrum ✓ GIK '17
- correlation functions of long string vertex operators match $T\bar{T}$ answer (w=1) Cui, Shu, Song, Wang '23
- spectrum of deformed discrete states & correl. functions do not match

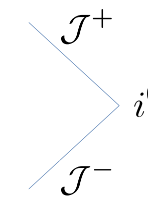
"stronger form" : UV theory shares certain universal features with single-trace $T\bar{T}$ – deformed CFTs

- black hole entropy $S(E)$ agrees with $T\bar{T}$ entropy (Cardy → Hagedorn) ✓ GIK '17
- the asymptotic symmetries of the ALD background are identical to those of single-trace $T\bar{T}$

  → same non-linear modification of Virasoro algebra in Fourier basis
- bnd. conditions on allowed diffeos dictated by black hole solutions $\omega(\mathcal{L}_{\xi^{ASG}}, \delta M) = \omega(\mathcal{L}_{\xi^{ASG}}, \delta J) = 0$

Lessons for flat holography?

- Penrose diagrams of ALD and AF spacetimes similar → similar holographic prescriptions?



- NS5 decoupling limit → the dual LST lives at i^0
- should the holographic dual to flat space similarly live at (the resolved) i^0? Marolf '06
- is the expected structure of correlation functions similar?
- if so, then could the solvable $T\bar{T}$ modeling of the ALD background give a clue of the "QFT structure"
- to look for? (note, in particular, that $T\bar{T}$ has a dimensionful coupling constant)

Conclusions

- in understanding non-AdS holography, some information about the "QFT structure" of the dual theory may be needed, in addition to symmetries, correlators, entropy

- may sometimes be obtained specific realisations of non-AdS holography within string theory

- dual theories were nonlocal and exhibited special structures not easily visible from standard observables or symmetries

- ASG analyses need careful interpretation (e.g. Kerr/CFT Virasoro $\not\rightarrow$ local 2d CFT !)

- a better understanding of the types of QFTs that may appear could help understand what is possible

What is the codimension of the holographic dual to flat space?

Thank you !

The primary condition

- main idea: use interplay of the two sets of symmetry generators

$$\begin{cases} \tilde{L}_n^\mu = R\, L_n - \lambda H_R J_n + \frac{\lambda^2 H_R^2}{4}\delta_{n,0}\,, & \tilde{J}_n^\mu = J_n - \frac{\lambda H_R}{2}\delta_{n,0} \\ \tilde{\bar{L}}_n^\mu = R_v \bar{L}_n - \lambda :H_R \bar{J}_n: + \frac{\lambda^2 H_R^2}{4}\delta_{n,0}\,, & \tilde{\bar{J}}_n^\mu = \bar{J}_n - \frac{\lambda H_R}{2}\delta_{n,0} \end{cases}$$

assumed
full quantum

- algebra LM (L_n, J_n) : Virasoro-Kac-Moody ; algebra RM $(\bar{L}_n, \bar{J}_n)$: non-linear modification of Vir.-KM

- LM: operators should be primary w.r.t. L_n, J_n ← implement conformal & affine U(1) transf.

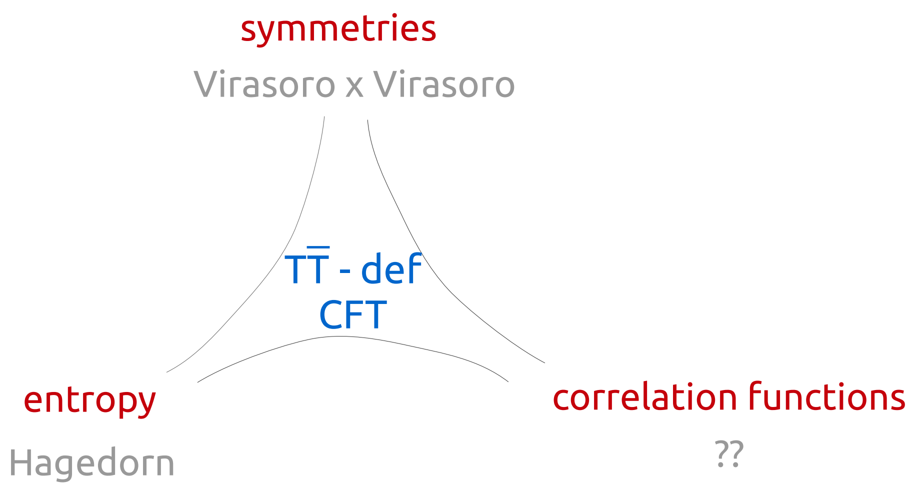
Ward id: $$[L_n, \mathcal{O}(w)] = e^{nw}(nh\mathcal{O} + \partial_w \mathcal{O})_{n\geq -1} \quad \text{w/} \quad h = \tilde{h} + \lambda\bar{p}\tilde{q} + \frac{\lambda^2\bar{p}^2}{4}$$

- introduce auxiliary ops. $\tilde{\mathcal{O}}(w,\bar{w})$ defined via $\partial_\lambda \tilde{\mathcal{O}}(w,\bar{w}) = [\mathcal{X}_{J\bar{T}}, \tilde{\mathcal{O}}(w,\bar{w})]$ ← identical correlation functions and Ward identities w.r.t. $\tilde{L}_n$ etc., as the operators in the undeformed CFT

$$\mathcal{O}(w,-) = e^{Aw} e^{\lambda\bar{p}\sum_{n=1}^\infty e^{nw}\tilde{J}_{-n}} \tilde{\mathcal{O}}(w,-) e^{-\lambda\bar{p}\sum_{n=1}^\infty e^{-nw}\tilde{J}_n} \times RM$$

Conclusions

- we have shown that, despite their non-locality, TT – deformed CFTs posess infinite symmetries
- various perspectives: abstract QM, classical Hamiltonian, Lagrangian, holographic + single-trace
- we have shown that the asymptotic symmetries of the asymptotically linear dilaton background in string theory are precisely those of single-trace $T\bar{T}$ – deformed CFTs
- this further suggests the relevant “QFT structure” for these bckgnds is closely related to that of



- a better understanding of both field theory and gravity (both doable!) may pave the way for precision holography in this background

Setup

- as we anticipate field-dependent symmetries → turn on non-trivial background to see this
- we thus consider the asymptotically linear dilaton black hole backgrounds
- $ALD \times S^3 \times T^4$ → use consistent truncation to 3d $ds^2 = ds_3^2 + \ell^2 ds_{S_3}^2$, $H = 2\ell^2\, \omega_{S^3} + b\, e^{2\phi}\, \omega_3$

$$d\bar{s}^2 = \frac{r^2}{\alpha' r^4 + \beta r^2 + \alpha' L_u L_v}\left(r^2 dU dV + L_u dU^2 + L_v dV^2 + \frac{L_u L_v}{r^2} dU dV\right) + k\frac{dr^2}{r^2}$$

$$e^{2\bar{\phi}} = \frac{k r^2}{\alpha' r^4 + \beta r^2 + \alpha' L_u L_v} \qquad \beta = \sqrt{p^2 + 4\alpha'^2 L_u L_v}$$

- classified linearized perturbations of this background: pure diffeos + propagating
- allowed diffeos : their symplectic form with the allowed modes, notably $\delta L_{u,v}$ must vanish

  → charge conservation

The “QFT structure” of solvable irrelevant deformations

- study “QFT structure” explicitly for Smirnov-Zamolodchikov deformations ($T\bar{T}$, $J\bar{T}$) ← exactly solvable irrelevant def’s

- e.g. for $J\bar{T}$ – deformed CFTs $\underbrace{SL(2,\mathbb{R})_L}_{\text{local \& conformal}} \times \underbrace{U(1)_R}_{\text{non-local}}$ (`half’ non-local)

symmetries

Virasoro x Virasoro

$J\bar{T}$ - def CFT

same structure as Kerr/“CFT”!

entropy

universal ~~Cardy~~

correlation functions

2&3 pf fixed by $\Delta(\bar{p})$

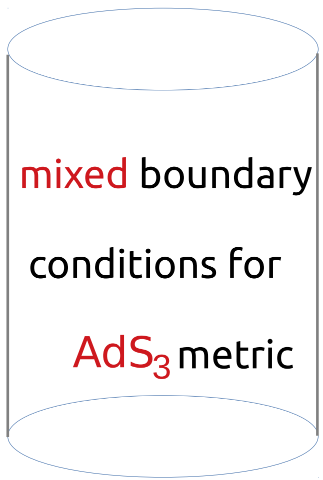
- $\exists$ non-local analogues of primary operators whose correlators are entirely determined by seed CFT

Holographic dual of $T\bar{T}$ - deformed CFTs

- $T\bar{T}$ deformation : **double trace**

- seed CFT : large c, large gap

  → Einstein gravity + low-lying matter fields



$$g^{(0)}_{\alpha\beta} - \mu g^{(2)}_{\alpha\beta} + \frac{\mu^2}{4} g^{(2)}_{\alpha\gamma} g^{(0)\gamma\delta} g^{(2)}_{\delta\beta} = \text{fixed}$$

MG, Monten '19

pure gravity $\approx$ Dirichlet at $\rho = -\mu$

McGough, Mezei & Verlinde '16

- holographic dictionary derived from field theory using Hubbard-Stratonovich trick

- 1[st] instance of mixed bnd. cond. on AdS_3 metric

  → bulk & boundary have independent definitions

  → contrast standard situation where properties of the boundary theory are inferred from the bulk

- change bnd. conditions on AdS_3 metric → radical modification of the bnd. theory: local → non-local

- precision holography

  → perfect match of bulk/boundary spectrum ✓

  → symmetries ✓

  → other observables?